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Nonlinear Bistable Detectors and Arrival-time Estimators Based on Parameter-tuning Stochastic Resonance

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14. ABSTRACT
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Nonlinear Bistable Detectors and Arrival-time Estimators Based on Parameter-tuning Stochastic Resonance

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Abstract— Parameter-tuning stochastic resonance (PSR) technique provides a new approach for signal processing. This paper will first fill the gap in the performance analysis of the nonlinear PSR-based detector by comparing it with the matched filter detector under both ideal conditions (white Gaussian noise, and perfect synchronization) and non-ideal conditions (colored noise, desynchronization, and low sampling rate) to identify its strengths and weaknesses. Then an innovative arrival-time estimator based on parameter-tuning stochastic resonance is proposed. It is shown that this PSR-based estimator will have better performance than the arrival-time maximum likelihood estimator under the conditions of colored noise and a short sampled data set. Also, a new detector which uses this PSR-based estimator for synchronization is proposed. The performance of this detector which combines the advantages of both the nonlinear PSR-based detector and the matched filter detector will surpass that of the matched filter detector using maximum likelihood estimators for synchronization. This paper reveals the potential applications of the parameter-tuning stochastic resonance technique in solving the real-world signal processing tasks.

Index Terms— Nonlinear detection, Nonlinear estimation, Nonlinear filters, Stochastic systems

I. INTRODUCTION

Usually, noise is thought to be annoying. In certain nonlinear systems, an extra amount of noise introduced into the systems will benefit the improvement of system performance. This phenomenon, in term of stochastic resonance (SR), was first put forward by Benzi in 1981 [1]. Since then, it has been widely applied into

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different areas, such as physics, biology, and engineering systems [2]. For example, the balance control of elderly people can be improved by noise [3], and noise can enhance the speech understanding of profoundly deaf people [4]. In signal processing areas, stochastic resonance technique has found applications in signal detection [5], signal transmission [6], and signal estimation [7]. For traditional stochastic resonance, the stochastic resonance effect is realized by adding an optimal amount of additional noise into the systems [2]. Tuning system parameters without adding noise, in term of parameter-tuning stochastic resonance, is another interesting approach to synchronize the input signal and noise [8]-[12][19][20]. Parameter-tuning stochastic resonance has been used in both analog and digital signal processing, such as the recovery of noisy multi-frequency signals [9], and the transmission of baseband binary pulse-amplitude modulated (PAM) signals under white or colored noise [11][12]. Also, the bit-error rate (BER) of the PAM signal transmission can be further reduced by combining PSR and error-correcting coding (ECC) [13][14]. In the transmission of PAM signals, a nonlinear bistable dynamic system whose parameters are tuned properly according to parameter-tuning stochastic resonance is used as a nonlinear detector to convert the noisy input signal back to the binary PAM signal. This nonlinear detector based on parameter-tuning stochastic resonance provides a new and effective approach to transmit the binary PAM signals. Also, we know that matched filter detectors (MFD) which are considered to be optimal under certain conditions are commonly used in real-world applications. It will be interesting to identify the strengths and weaknesses of both detectors in order to provide a guideline to apply them in real-world applications. This paper will first fill the gap in the performance analysis of the nonlinear PSR-based detector by comparing it with the matched filter detector under both ideal conditions (white Gaussian noise, and perfect synchronization) and non-ideal conditions (colored noise, desynchronization, and low sampling rate). It will also be interesting to explore the potential applications of the parameter-tuning stochastic resonance technique in which the approaches based on PSR can

provide competitive or even better performance than the techniques commonly used today. This paper will then propose an innovative arrival-time estimator based on parameter-tuning stochastic resonance, and also a new detector using this estimator for synchronization. It is shown that they will have better performance than the arrival-time maximum likelihood estimators (MLE) and matched filter detectors using MLE for synchronization respectively under the conditions of colored noise and a short sampled data set. Under these conditions, the input signal is totally buried in the background noise. This will cause the dramatic performance degradation of MLE. For the arrival-time estimator based on PSR, the signal-location information can still be preserved because of the special characteristics of the PSR-based pre-processor adopted. This paper reveals the possibility of applying parameter-tuning stochastic resonance in solving real-world signal processing tasks.

The rest of this paper is organized as follows. Section II first introduces the nonlinear detectors based on parameter-tuning stochastic resonance without using arrival-time estimators. Then the performance comparisons with the detectors based on matched filters are performed under both ideal conditions (white Gaussian noise (WGN), and perfect synchronization), and non-ideal conditions (desynchronization, colored noise, and low sampling rate) in order to investigate this detector's strengths and weaknesses and identify the opportunities for further improvement. In Section III, the arrival-time estimator based on parameter-tuning stochastic resonance is proposed. Also, its performance is compared with the related maximum likelihood estimator. A new detector with this arrival-time estimator will be proposed in Section IV. Finally, Section V closes the paper with brief concluding remarks and future research directions.

II. NONLINEAR BISTABLE DETECTORS AND COMPARISONS WITH MATCHED FILTER DETECTORS

The nonlinear bistable double-well dynamic system can be described as [9]

$$\dot{x}(t) = ax(t) - bx^3(t) + s(t) + \eta(t), \qquad (1)$$

where a and b are system parameters, $s(t)$ is the input signal, and $\eta(t)$ is additive white Gaussian noise with zero mean average and autocorrelation of $\langle \eta(t)\eta(0) \rangle = 2D\delta(t)$.

For traditional stochastic resonance, the chosen performance measures, such as output signal-to-noise ratio (SNR), will be maximized when an optimal amount of additional noise is added into this system [2]. For parameter-tuning stochastic resonance, the signal-to-noise ratio will be maximized by tuning the values of

system parameters a, and b without adding additional noise into the system. This scheme has been used to recover the multi-frequency signals corrupted by noise [9].

This paper will focus on the baseband binary PAM signal processing which is expressed as

$$s(t) = s_1(t) = A \quad or \quad s(t) = s_2(t) = -A,$$
$$for \quad (n-1)T_b < t \le nT_b, \quad n = 1, 2, ..., \qquad (2)$$

where A is the signal amplitude, and T_b is the bit duration.

System (1) can also be used to transmit signal (2) corrupted by noise as a nonlinear filter [11]. The noisy input signals can be recovered back to binary signals based on the detection decisions made according to the values of system output $x(t)$ at the sampling points of nT_b. The performance of this receiver based on (1) can be measured by bit-error rate which will be minimized by tuning system parameters a and b to their optimal values [11]. In this paper, we will call this receiver as the nonlinear bistable detector, or PSR-based detector (PSRD).

The theoretical foundation of parameter-tuning stochastic resonance is the derivation of the Fokker-Planck equation (FPE) satisfied by the dynamic probability density function of system output $x(t)$. For system (1), its FPE is derived as [11]

$$\frac{\partial \rho(x,t|s_i)}{\partial t} = -\frac{\partial}{\partial x}[C(x)\rho(x,t|s_i)] + D\frac{\partial^2 \rho(x,t|s_i)}{\partial x^2}, \quad (3)$$

where $i = 1, 2$, $C(x) = ax - bx^3 + s_i$, and $\rho(x,t|s_i)$ is the dynamical probability density function of $x(t)$ when $s_i(t)$ is transmitted.

From the stationary solution $\rho(x)$ and the expanding solution $\rho(x,t)$ of (3), the output bit-error rate of the PSR-based detector can be defined as [18]

$$BER = P_{s_1}P(s_2|s_1) + P_{s_2}P(s_1|s_2), \qquad (4)$$

where P_{s_i} is the probability of transmitting symbol $s_i(t)$, $P(s_2|s_1)$ and $P(s_1|s_2)$ are the probability of error detection which can be defined as

$$P(s_2|s_1) = \frac{1}{2}\int_0^\infty \rho(x,T_b|A)dx + \frac{1}{4}\int_0^\infty \rho(x,2T_b|A)dx$$
$$+ \frac{1}{8}\int_0^\infty \rho(x,3T_b|A)dx, \qquad (5)$$

$$P(s_1|s_2) = \frac{1}{2}\int_{-\infty}^0 \rho(x,T_b|-A)dx + \frac{1}{4}\int_{-\infty}^0 \rho(x,2T_b|-A)dx$$
$$+ \frac{1}{8}\int_{-\infty}^0 \rho(x,3T_b|-A)dx. \qquad (6)$$

The bit-error rate is a function of system parameters a and b, it will be minimized by tuning these system parameters optimally. This is the concept of parameter-tuning stochastic resonance. Figure 1 shows the system
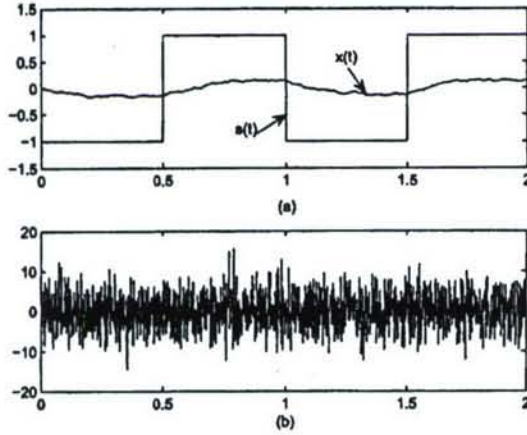
Fig. 1. (a) Input $s(t)$, and System Output $x(t)$ (b) Noise $\eta(t)$

TABLE I
COMPARISONS BETWEEN PSRD AND MFD UNDER IDEAL
CONDITIONS

Noise Intensity D	BER of MFD	BER of PSRD
0.025	0.03	0.11
0.05	0.086	0.172
0.25	0.272	0.337

output $x(t)$ for input signal $s(t)$ corrupted by noise $\eta(t)$ when the system parameters are tuned properly. It is obvious that the noise is greatly suppressed. This will, in turn, reduce this PSR-based detector's probability of error detection.

We have also demonstrated that the bit-error rate can be further reduced by combining the PSR with ECC [13][14]. The binary input is encoded before transmission, and the decoder is added after the PSR-based detector. ECC has the ability to detect and correct certain number of bit errors using the inserted redundant bits.

The nonlinear bistable detector based on parameter-tuning stochastic resonance is an innovative and effective approach for signal processing. In this area, it is also well known that the PAM signal (2) corrupted by noise can also be detected by matched filter detectors [16]. If the noise is white Gaussian noise, the Neyman-Pearson (NP) criterion and the maximum signal-to-noise ratio criterion both lead to matched filter detectors, which are optimal linear detectors in this case [16]. The matched filter has an impulse response of time-reversed version of the input signal

$$h(t) = s(T_b - t), \quad t \in [0, T_b], \tag{7}$$

where $s(t)$ is the input signal, and T_b is the bit-duration of the input signal.

It will be interesting to first compare this PSR-based detector with the matched filter detector to identify its strengths and weaknesses. This will provide us the clues for the further improvement of its performance. In order to get a whole picture, the comparisons will be performed under both ideal conditions and non-ideal conditions.

First, we will compare it with MFD under ideal conditions (WGN, and perfect synchronization). Of course,

the performance of this PSR-based detector cannot be better than MFD in these cases. Here, we just want to know how close its performance will approach that of the optimal detector by simulations. Table I shows the bit-error rates reached by both detectors for different input noise intensity, when $T_b = 0.1$, and sampling period $T_s = 0.001$. From it, we can notice that MFD is indeed the optimal detector under ideal conditions.

Now, we will compare them under non-ideal conditions. The matched filter detectors require the perfect synchronization in order to maintain their optimal performance. The perfect synchronization means the signal arrival time is known and the matched filter detector will make decisions by sampling at $t = T_b$. As shown in [16], the performance of matched filter detectors will be degraded if the signal arrival time is unknown, or the signal does not begin at $t = 0$. If the noise is WGN, the performance comparisons under the condition of desynchronization have already been mentioned in [17][18]. It was shown that the nonlinear bistable detectors are more robust to the desynchronization than the matched filter detectors. These comparisons will not be repeated here. In this paper, we will conduct the comparisons, when the noise is colored.

The fact that the matched filter detector is an optimal detector is also under the assumption of white noise. If the noise is colored, the matched filter detectors will not be optimal, and the matched filters cannot maximize the SNR either. Prewhitening filters are the common techniques used to maintain the performance of matched filter detectors in colored noise. This, however, requires the exact knowledge of the noise's power spectral density, which is often impossible in real-world applications.

If the colored noise is wide-sense stationary (WSS), it can be approximated by the following exponentially correlated colored noise with Gaussian distribution [12][15]

$$\frac{d\eta}{dt} = -\frac{1}{\tau_1}\eta + \frac{1}{\tau_1}\Gamma(t), \tag{8}$$

where τ_1 is the correlation time of the noise, $\Gamma(t)$ is white Gaussian noise with zero mean and the autocorrelation function

$$\langle \Gamma(t)\Gamma(0) \rangle = 2D\delta(t), \tag{9}$$

where D denotes the noise intensity.

For colored noise described by (8), the Fokker-Planck equation of system (1) can be derived approximately in a method similar to that introduced in [12]

$$\frac{\partial \rho(x,t|s_i)}{\partial t} = -\frac{\partial}{\partial x}[C(x)\rho(x,t|s_i)]$$
$$+D\frac{\partial^2}{\partial x^2}[\frac{1}{1-\tau_1 C'(x)}\rho(x,t|s_i)], \quad (10)$$

where $i = 1, 2$, $C(x) = ax - bx^3 + s_i$, and $\rho(x,t|s_i)$ is the dynamical probability density function of $x(t)$ when $s_i(t)$ is transmitted.

Similar to the white Gaussian noise case, the bit-error rate defined by (4) can be expressed by the stationary and expanding solutions of (10). It will be minimized by optimally tuning the system parameters a and b. This reveals that the nonlinear bistable detector based on parameter-tuning stochastic resonance can also be applied to colored noise cases.

If the noise information is unavailable, noise spectral estimators have to be adopted in order to apply matched filters in colored noise situations. The estimation will degrade the performance of matched filters, especially for a short sampled data set. For the nonlinear bistable filters based on PSR, we can assume that the system parameters can be optimally adjusted adaptively to minimize the bit-error rate without the knowledge of noise. For example, the adaptive mechanism can be realized in two methods. The first method will utilize a bank of PSR-based filters with pre-tuned system parameters. The one with minimal bit-error rate will be chosen as the detector to process the received signals. A pilot signal will be used in order to calculate the bit-error rate. For the second method, a pilot signal will also be used. It will be transmitted first. The system parameters will then be tuned on-line to minimize the bit-error rate for this pilot signal. After this adjustment, the real signal can be transmitted. Assuming the colored noise is totally unknown, it will be hard to convert it to white Gaussian noise to maintain the matched filter's optimal performance. In this case, it would be interesting to compare the performance of the PSR-based nonlinear detectors whose system parameters are tuned optimally with that of matched filter detectors without using prewhitening filters. Table II shows the comparison results, when $\tau_1 = 0.1$, $T_b = 0.1$, and $T_s = 0.001$. It is obvious that matched filter detectors still have better performance than PSR-based detectors. The performance of PSR-based detectors, however, is very close to that of matched filer detectors in this case.

Now, we assume that the non-ideal conditions include both the desynchronization and colored noise. In this

TABLE II
COMPARISONS BETWEEN PSRD AND MFD UNDER CONDITION OF COLORED NOISE

Noise Intensity D	BER of MFD	BER of PSRD
0.025	≈ 0	≈ 0
0.05	0.011	0.065
0.25	0.143	0.17
0.5	0.243	0.257

TABLE III
COMPARISONS BETWEEN PSRD AND MFD UNDER CONDITIONS OF COLORED NOISE AND DESYNCHRONIZATION

Desynchronization	BER of MFD	BER of PSRD
1%	0.246	0.255
2%	0.25	0.257
5%	0.262	0.258
15%	0.306	0.261

case, $P(s_2|s_1)$ and $P(s_1|s_2)$ are re-defined as

$$P(s_2|s_1) = \frac{1}{2}\int_0^\infty \rho(x,T_b-t_0|A)dx + \frac{1}{4}\int_0^\infty \rho(x,2T_b-t_0|A)dx$$
$$+\frac{1}{8}\int_0^\infty \rho(x,3T_b-t_0|A)dx, \quad (11)$$

$$P(s_1|s_2) = \frac{1}{2}\int_{-\infty}^0 \rho(x,T_b-t_0|-A)dx$$
$$+\frac{1}{4}\int_{-\infty}^0 \rho(x,2T_b-t_0|-A)dx + \frac{1}{8}\int_{-\infty}^0 \rho(x,3T_b-t_0|-A)dx.(12)$$

where t_0 is the desynchronization, $\rho(x,t|A)$, and $\rho(x,t|-A)$ are the solutions of (10).

The performance comparisons under these conditions are shown in Table III, when $\tau_1 = 0.1$, $T_b = 0.1$, $T_s = 0.001$, and $D = 0.5$. According to [17][18], the performance of nonlinear bistable detectors will surpass that of matched filter detectors if there is a 20% or more desynchronization under the white Gaussian noise condition. From Table III, we can notice that this value is reduced to about 5% under the colored noise condition. This means that it would be much easier for the PSR-based detectors to obtain better performance than that of matched filter detectors.

It is also worth noting that the PSR-based detectors are more robust to the sampling rate than matched filter detectors. If T_s is changed from 0.001 to 0.005, there will be only a 0.1% change of the BER for the PSR-based detectors, compared with a 1% change for the matched filter detectors under the same conditions. The low sampling rate will also further degrade the performance of MFD. For example, the BER for MFD is 0.249 and BER for PSRD is 0.184, when $T_b = 0.1$, $T_s = 0.01$, $\tau_1 = 0.1$, $D = 0.2$, and the desynchronization is 20%.

III. ARRIVAL-TIME ESTIMATORS BASED ON PSR

From Section II, we know that the matched filter detectors will yield poor detection performance with unknown signal arrival time. This is also true for PSR-based detectors, even though they are more robust than matched filter detectors. In order to circumvent this problem, arrival-time maximum likelihood estimators are commonly used together with matched filter detectors. The MLE has the properties of being asymptotically unbiased, and asymptotically achieving the Cramer-Rao lower bound (CRLB). It is asymptotically efficient. For a large data set, the arrival-time MLE can be taken as an optimal unbiased estimator.

We assume the input signal with unknown arrival time n_0 is corrupted by white Gaussian noise

$$x[n] = s[n - n_0] + w[n], n = 0, 1, ..., N - 1, \quad (13)$$

where $s[n]$ is a known deterministic input signal with nonzero vales over the interval $[0, M-1]$, and $w[n]$ is white Gaussian noise. The observation period $[0, N-1]$ should cover the signal interval $[n_0, n_0 + M - 1]$ for all possible n_0.

The $\hat{n}_0$ which is the MLE of n_0 can be calculated by [15]

$$arg \max_{n_0} \{ \sum_{n=n_0}^{n_0+M-1} x[n]s[n - n_0] \}. \quad (14)$$

The performance of arrival-time MLE estimator (14) which is derived under the condition of WGN will be affected by the length of the data set, the noise property, and also the signal-to-noise ratio. The nonlinear filter (1) based on parameter-tuning stochastic resonance has the ability to increase the signal-to-noise ratio by suppressing the noise [9]. We, therefore, propose a new arrival-time estimator by combing PSR and MLE together. In this estimator, the PSR-based filter (1) is used as a pre-processor of MLE (14). The purpose is to improve this estimator's performance under non-ideal conditions, such as colored noise, and a short sampled data set. We call this innovative arrival-time estimator as PSR based estimator (PSRE).

It is interesting to investigate the performance of this PSR-based estimator by comparing it with the original MLE (14) under both ideal and non-ideal conditions.

Table IV shows the performance comparisons of these two estimators under ideal conditions (WGN, and a large data set), when $T_b = 0.1$, and $n_0 = 50$. From this table, it is verified that the original MLE (14) is better under ideal conditions. The PSR-based estimator, however, has the performance very close to the optimal one. The reason why PSRE is a little worse than MLE (14) is that the

TABLE IV
COMPARISONS BETWEEN PSRE AND MLE UNDER IDEAL CONDITIONS

Noise Intensity	Mean		Variance	
	MLE	PSRE	MLE	PSRE
0.005	50	54	0	0
0.025	50	54	0	0
0.05	50	53	0	0
0.25	56.66	65.3	995.6	1487.1

pre-processor might change the noise characteristics and cause it no longer to be white Gaussian noise. This will degrade the performance of the MLE connected to it, even though the PSR-based pre-processor might amplify the SNR.

Under ideal conditions, there seems to be no reason to apply the PSR technique together with MLE (14). The MLE (14), however, is asymptotically efficient, which needs a large sampled data set in order to maintain the optimal performance. Also, MLE (14) is derived under the assumption of white Gaussian noise. For colored noise, it will be difficult to derive the arrival-time MLE, because of the unavailability of an exact expression of the probability density function (PDF) of the sampled data. The MLE (14) will not guarantee to be optimal for colored noise. If the noise is WSS and can be approximated by the autoregressive (AR) model of order one, the PDF of the sampled data can be derived approximately [16]. This, however, also needs a large data record. So, the arrival-time MLE (14) will obtain degraded performance under these non-ideal conditions. It will be interesting to explore whether the PSR-based filter (1) can be of any help with the performance improvement in these cases. Similarly, if the noise is totally unknown and the sampled data set is short, the original MLE (14) without prewhitening filters will be used for the performance comparisons with the PSRE in these non-ideal conditions. For the PSR-based pre-processor, the adaptive mechanism introduced in Section II will also be used to tune the system parameters optimally in these cases.

First, we assume the conditions of a short sampled data set, and white Gaussian noise. The results are shown in Table V, when $T_b = 0.1$, and $n_0 = 50$. From this table, we notice that both MLE (14) and PSRE still have good performance, and MLE (14) is a little better than the PSRE.

Then, we perform the comparisons under the conditions of a large sampled data set and colored noise. The results are shown in Table VI, when $T_b = 0.1$, $\tau_1 = 0.1$, and $n_0 = 50$.

Noise Intensity	Mean		Variance	
	MLE	PSRE	MLE	PSRE
0.005	50	54	0	0
0.025	49.6	53.9	0.33	0.9
0.05	49.9	54.5	0.64	135
0.25	65.1	70.3	1254	1488

Noise Intensity	Mean		Variance	
	MLE	PSRE	MLE	PSRE
0.005	50	52	0	0
0.025	50	64	0	0
0.05	50	63	0	0
0.25	50	61	0	0

It seems that both MLE (14) and PSRE have very good performance with colored noise. The reason is that the average noise intensity is decreased, because of the noise correlation time τ_1. The colored noise described by (8) has the power spectral density [15]

$$S(\omega) = \frac{2D}{1 + \omega^2 \tau_1^2}. \qquad (15)$$

Up to now, we have demonstrated that MLE (14) and PSRE have good and close performance. Now, we will investigate their performance under the conditions of a short sampled data set and colored noise. The results are shown in Table VII, when $T_b = 0.1$, $T_s = 0.01$, $\tau_1 = 0.1$, $n_0 = 50$, and the signal length $L = 5$ bits. From this table, we can notice that the performance of both estimators will degrade dramatically, when the noise intensity is increased to certain values. In these cases, both estimators can hardly provide useful information.

It is expected that MLE (14) will have poor performance under these non-ideal conditions. For PSR-based estimators, the nonlinear pre-processors should be able

Noise Intensity	Mean		Variance	
	MLE	PSRE	MLE	PSRE
0.005	50	52	0	0
0.025	50	52	0	0
0.05	50	52	0	0
0.5	1256	2479	1.6×10^5	2.6×10^5

to help the performance improvement, if certain kind of synchronization between the input signal and noise can still occur by tuning the system parameters under the conditions when MLE (14) could not estimate the arrival-time. In this case, the characteristics of the nonlinear bistable dynamic system (1) will cause the values of output $x(t)$ to change between the two stable states of system (1) continuously at the input signal frequency as shown in Figure 1, if we assume that the signal $s(t)$ which is used for the arrival-time estimation is periodic (square wave in this case). The stochastic resonance effect, however, cannot be realized even when the system parameters are tuned to their optimal values, if the noise intensity is too large. This means the bit-error rate (4) cannot be reduced under certain level. In this case, PSR-based pre-processors cannot benefit the estimations. Fortunately, the conditions for the estimations to be beneficial from the PSR-based pre-processors do exist in reality. For example, the bit-error rate can be reduced under 25% according to (4) and (10), when $T_b = 0.1$, $T_s = 0.01$, $\tau_1 = 0.1$, and $D = 0.5$. From Table VII, we know MLE (14) cannot estimate the arrival-time at all under these conditions. The PSR-based pre-processors should be useful in these cases in which there will be about one bit-error among every five bits on average. This will provide useful information to locate the signals in the background noise. So, there must be some ways to further improve the performance of the PSR-based estimators. After careful investigation, the factors that cause the dramatic performance degradation of PSR-based estimators are identified. Methods are then adopted to overcome these problems. We notice that we cannot directly use MLE (14) together with PSR-based filter (1). MLE (14) estimates the arrival time n_0 by sliding the input signal $s(t)$ along the noisy sampled data set $x(t)$ and finding the location with the maximal similarity. The output $x(t)$ corresponding to $s(t)$, however, is deformed by the nonlinearity of system (1) and noise. We cannot locate the input signal by simply comparing the waveform of $x(t)$ with the original input signal $s(t)$. Other criteria must be adopted to replace (14). The bit-error rate is one option. Ideally, we can locate the section $[n_0, n_0 + M - 1]$ with the minimal bit-error rate and take n_0 as the arrival-time estimation. The reason is that the PSR-based pre-processor is tuned optimally to minimize the bit-error rate for the input signal $s(t)$. The bit-error rate corresponding to the pure background noise without input signals will be higher than that with input signals. For the example just mentioned, there will be about one bit-error among every five bits on average, if the system parameters are tuned properly. We can, therefore, locate the input signal with a length of five bits, if we can find

a range $[n_0, n_0 + M - 1]$ with no or one bit-error. This is illustrated in Figure 2, where $s(t)$ is the input signal with a length of five bits, $x(t)$ is the output of (1), and $y(t)$ is the binarized output of (1). The input signal can be located by the bit-error rate criterion in this case.

The bit-error rate alone, however, cannot locate the input signals in all cases. The reason is that the calculated bit-error rate (4) is an average value. The system parameters are tuned to minimize this average bit-error rate. There are still situations in which the individual bit-error rate is larger than this average value. Fortunately, the waveform of output $x(t)$ corresponding to the input signal still has some features for it to be different from that corresponding to pure background noise in these cases. This is shown in Figure 3. In this figure, we can notice that the bit-error rate of the output section related to the input signal $s(t)$ with a length of five bits is 2/5. It cannot be used to tell the input signal apart from the background noise. We can also notice, however, that the section of $x(t)$ corresponding to input signal $s(t)$ changes in a pattern different from that corresponding to pure background noise. These waveform patterns are caused by the periodic input signal and the bistable features of system (1). So, pattern matching will be used as the other criterion for the arrival-time estimation. We can gather the typical possible waveform patterns corresponding to input signals under the influence of noise. We will also give each waveform pattern a weight. The more accurately a pattern can be used to estimate the input signal location, the larger the weight value assigned to it will be. The largest value is one, the smallest value is zero. For example, we can assign the waveform pattern with zero bit-error rate a weight of one. Also, the output waveform corresponding to a square-wave input signal is not exact square. If the bit-error rate is zero, the output waveform will be similar to that shown in Figure 1. The shape of the waveform is affected by the response time λ of system (1). It can be calculated in the methods introduced in [9][12]. The pattern matching can be calculated by

$$\left| 1 - \frac{\sum_{n=n_0}^{n_0+M-1}(x[n] - \overline{x[n]})p[n-n_0]}{M\sqrt{\overline{(x[n] - \overline{x[n]})^2} \; \overline{p[n]^2}}} \right| \leq \varepsilon, \quad (16)$$

where $x[n]$ is the sampled output of the PSR-based filter (1), $p[n]$ is the sampled waveform pattern, and ε is a certain small positive constant. The overbar denotes the time average over the selected ranger $[n_0, n_0 + M - 1]$.

To put these together, we will combine the above two criteria into one criterion. We will also assign the bit-error rate of the output over the selected range $[n_0, n_0 + M - 1]$ a weight. Similarly, the more accurately a bit-error rate value can be used to estimate the input signal location, the larger the weight value assigned to it will be. For example, we can assign zero bit-error rate a weight value of one. The weight value will be zero, if every bit is incorrect. In this case, estimator (14) which

is used after the PSR-based pre-processor is replaced by the following estimator

$$\arg\max_{n_0}\{w_1 W_1 + w_2 W_2\}, \quad (17)$$

where w_1 and w_2 are the weight coefficient of these two criteria which can be adjusted properly in reality to increase the estimation accuracy, W_1 is the bit-error rate weight value, and W_2 is the pattern matching weight value. Both of them are calculated based on the output over the range $[n_0, n_0 + M - 1]$.

In most cases, estimator (17) which combines the above two criteria can tell the output section corresponding to the input signal apart from that corresponding to pure background noise. Now, we assume that pure background noise can, with a very low probability, cause both the bit-error rate and the waveform pattern to be similar to those caused by input signals. In this case, the confusion or the false estimations can be eliminated by repeating the estimation processes when possible. True estimations can repeat, while false estimations caused by pure noise cannot.

Estimator (17) used after the PSR-based pre-processor can be used to estimate the signal arrival-time under both ideal and non-ideal conditions. It can eliminate the fixed estimation bias under ideal conditions. This will ensure that PSR-based estimators will almost have the same performance as maximum likelihood estimators under ideal conditions. Moreover, estimator (17) will have better performance than MLE (14) under non-ideal conditions. For example, MLE (14) cannot, as shown in Table VII, be used to estimate the arrival-time, because it provides no useful estimation information, when $T_b = 0.1$, $T_s = 0.01$, $\tau_1 = 0.1$, $D = 0.5$, and $L = 5$. In this case, the PSR-based pre-processor, however, can still maintain a bit-error rate under 25% by tuning the system parameters properly. There is about one bit-error for every five bits. The condition of one bit-error for every five bits on average makes it still possible to estimate the signal arrival-time based on the criteria of both the bit-error rate and the pattern matching. This is shown in Figure 2 and Figure 3. This condition, however, will make it impossible to always estimate the input signal location exactly, because it is of possibility for the output section over the section $[n_1, n_1 + M - 1]$ to be similar to that over the section $[n_0, n_0 + M - 1]$ under this condition, when n_1 is very close to n_0. The estimation value $\hat{n}_0$ will be around the true signal location n_0. We can assume that the probability of $\hat{n}_0 > n_0$ and the probability of $\hat{n}_0 < n_0$ are equal, the average estimation value will then be n_0. This means the PSR-based estimator will be unbiased. Also, we can assume that the output section over the range $[n_0, n_0 + M - 1]$ can be distinguished
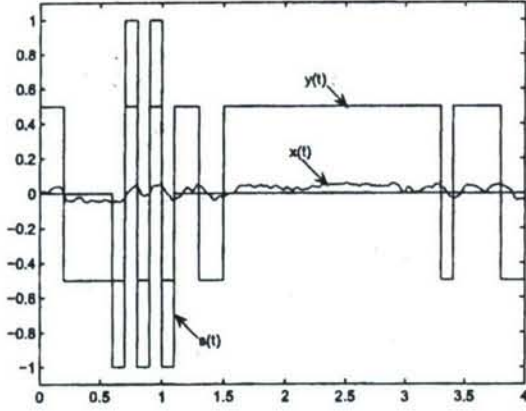
Fig. 2. Input Arrival-time Estimation with Bit-error Rate Criterion
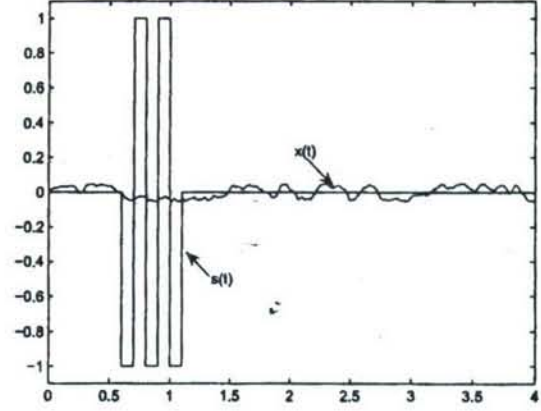

Fig. 3. Arrival-time Estimation with Pattern Matching Criterion

from the output section over the range $[n_1, n_1 + M - 1]$, when there is no overlap between these two sections, i.e., $|n_0 - n_1| > M$. In this case, the output section over the range $[n_0, n_0 + M - 1]$ will be influenced by both input signal $s(t)$ and noise $\eta(t)$, while the output section over the range $[n_1, n_1 + M - 1]$ will be totally affected by the pure background noise $\eta(t)$. So, the estimation variance of the PSR-based estimator is upper bounded by M^2 under this non-ideal condition, where $M = LT_b/T_s$. Compared with MLE (14), the PSR-based estimator can still be used for the arrival-time estimation with certain accuracy under these non-ideal conditions. The reason is that the input signal is totally buried in the background noise for MLE (14) under these conditions, while the output waveform patterns can still be identified for estimator (17).

It is also worth noting that the performance of PSR-based estimators can be further improved, if the error-correcting code can be applied together with the parameter-tuning stochastic resonance technique. Under the conditions mentioned in the above example, The error-correcting code can further reduce the bit-error rate of the nonlinear pre-processor based on parameter-tuning stochastic resonance by detecting and correcting certain number of error bits. This will, in turn, improve the arrival-time estimation accuracy based on the bit-error rate criterion.

IV. NONLINEAR BISTABLE DETECTORS WITH ARRIVAL-TIME ESTIMATORS

Under ideal conditions, there will be no need to adopt PSR-based detectors. This is shown in Section II. If there is desynchronization, the performance of both PSR-based detectors and matched filter detectors will degrade. Arrival-time estimators will be used to overcome this problem. Usually, arrival-time maximum likelihood estimators (14) are used together with the matched filter detectors to maintain the performance. This combination, however, still has poor performance under non-ideal conditions, because MLE (14) cannot estimate the arrival-time correctly. Now, we will propose a new detector which combines nonlinear bistable detectors, matched filter detectors together with the arrival-time estimators introduced in Section III. It is shown in Figure 4. In this proposed detector, the arrival-time estimator will use PSR-based estimator (17) rather than MLE (14), because PSR-based estimators have better performance under these non-ideal conditions. The detection part will use both the nonlinear bistable detector and the matched filter detector, because each has its own advantages and disadvantages as shown in Section II. The one with better performance will be chosen for the signal detection for each particular situation. The switch mechanism is implemented by choosing the one with a lower bit-error rate for a given pilot signal. The switch process can be performed periodically in order to be adapted to the changing environment. The new detectors which combine the advantages of both nonlinear bistable detectors and matched filter detectors will have better performance than matched filter detectors with maximum likelihood estimators. For example, assuming that the estimation accuracy of MLE (14) is 20%, and the estimation accuracy of PSR-based estimator is 5% under some non-ideal conditions, the nonlinear bistable detector will be chosen for the signal detection and this proposed detector will generate a bit-error rate of 0.258,
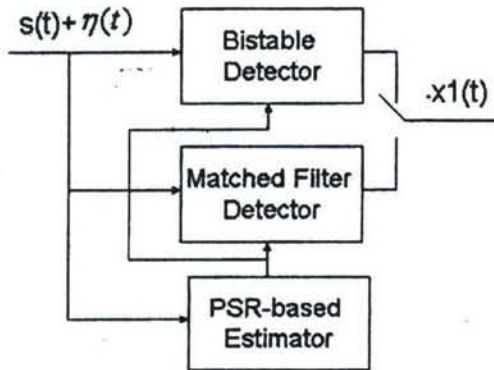
Fig. 4. Estimator Based on Parameter-tuning Stochastic Resonance

while the matched filter detector with MLE (14) will obtain a bit-error rate of 0.325, when $T_b = 0.1$, $\tau_1 = 0.1$, and $D = 0.5$.

V. CONCLUSION AND FUTURE WORK

This paper proposes a new PSR-based estimator of unknown signal arrival-time, and also a PSR-based detector. The performance comparisons with arrival-time maximum likelihood estimators and matched filter detectors are performed under both ideal and non-ideal conditions. It is revealed that the proposed estimators and the detectors based on parameter-tuning stochastic resonance will have better performance under the conditions of desynchronization, colored noise, and a short sampled data set. Our future work will focus on exploring their applications in solving real signal processing tasks, and also comparing them with matched filter detectors with prewhitening filters (assuming the availability of noise power spectral density) and other techniques commonly used in real applications under wider conditions, such as non-Gaussian noise. Our future work will also be directed at investigating the applications of parameter-tuning stochastic resonance in other signal processing areas.

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